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Mechanics Of Muscle Contraction Part Iii: Isokinetic Shortening Contraction

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Abstract

In this paper we applied a proposed theory (Tsili, 2013a) and we investigated the isokinetic shortening contraction of the sarcomer. We assumed that during contraction, the change of its cross-section area was negligible. Our model for special values of parameters, predicts the contraction speed measured by Faulkner et., al., (1986) at temperature 37oC. Also our findings that deal with nor-malized stress, force, length and volume of the sarcomer, are very closed to the results derived from statistical analysis on “raw” data taken from Gordon et., al., (1966); Spector et., al., (1980); Roy et., al., (1982); Powel et., al., (1984); Faulkner et., al., (1986);Pollack (1990); Epstein and Herzog (1998); Burkholder et., al., (2001); Makropoulou, (2009).

INTRODUCTION

The purpose of this paper is to study the problem of iso-kinetic shortening contraction of the sarcomer. For that reason we will base upon a recently developed theory (Tsili, 2013a). The main assumption of this work is that during contraction the change of cross-section area of the sarcomer is small and could be neglected.

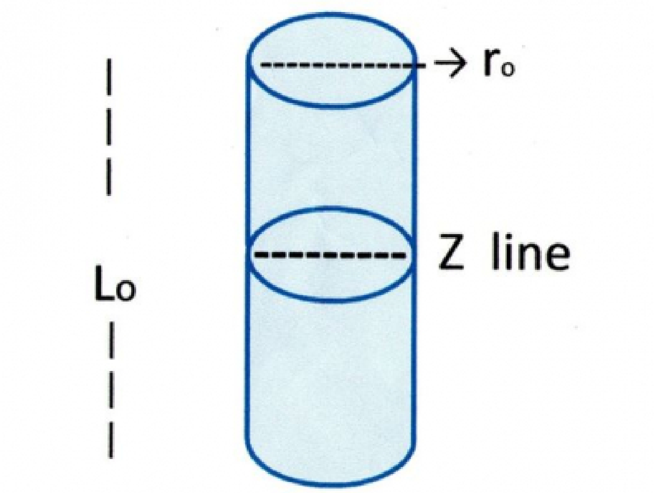
THE PHYSICAL APPROXIMATION OF THE PROBLEM

The basic equations of our theory are in our previous paper (Tsili, 2013a) and they will not be repeated here. We supposed that at t <0, the sarcomer was free of stress. In resting state it had a length Lo, a radius r_o and a temperature [illegible], as indicated in Fig. 1. Consequently its cross -section area and initial volume of sarcomer were S_o and [illegible], respectively, where:

$S_o = \pi r_o^2$ and $[illegible]_o = S_o.L_o$ (2.1)

Figure 1

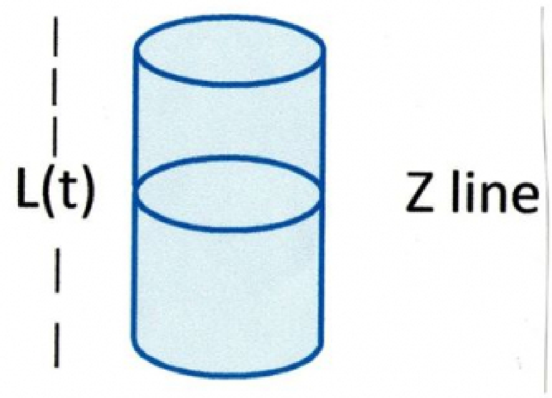
In resting state the sarcomer had a length Lo and a radius So .



We assume that at t=0 the motor neuron sends a stimulative pulse to the motor unit, in order to start the process of muscle contraction. Due to the delay of the response, at t1 >0 a mechanical stress is produced. The last simultaneously displaces with constant speed upwards and downwards the material particles of the sarcomer that lie below and above the z line respectively. As a result of the above displacements, the sarcomer shortens (see Fig. 2.). The process of shortening continues until a time t_s. At $t > t_s$ the contraction stops and the inverse process of relaxation starts.

Figure 2

The sarcomer at time t ≥ t1, during shortening. Accordingly to all that we stated in previous paragraph, the displacements are: ur= uθ = 0 and uz = aθοtz (2.2)1-2-3



Accordingly to all that we stated in previous paragraph, the displacements are:

$u_r = u_\theta = 0$ and $u_z = a\theta_o tz$ (2.2)1-2-3

where a is an unkown constant. Therefore the velocity of the material particle of sarcomer is:

$v_r = du_r/dt = 0$ $v_\theta = du_\theta/dt = 0$ and $v_z = du_z/dt = a\theta_o z$ (2.3)1-2-3

Then strain–displacement equations become:

$E_{rr} = E_{\theta\theta} = E_{r\theta} = E_{z\theta} = E_{rz} = 0$ and $E_{zz} = a\theta_o t$ (2.4)1-2-3-4-5-6

Substituting the above into stress - strain equation (see Tsili, 2013a) it follows that all stresses vanish:

$T_{rr} = T_{\theta\theta} = T_{\theta z} = T_{rz} = T_{r\theta} = 0$ (2.5)1-2-3-4-5

except from the axial stress:

$T_{zz} = c_{33} a\theta_o t + a_z + a_{zz} a\theta_o$ (2.6)

CALCULATION OF STRESS, FORCE, LENGTH AND VOLUME OF THE SARCOMER, USING “RAW” DATA

Spector et., al., (1980); Roy et., al., (1982); Powel et., al., (1984) calculated the maximal isometric stress P_o:

$P_o = 225 KN/m^2$ (3.1)

Accordingly to Gordon et., al., (1966) and to Pollack (1990), the initial length and diameter of sarcomer are :

$L_o = 2.22$ μm and $d_o = 2r_o = 1$ μm $(3.2)_{1\text{-}2}$ respectively (1μm=10^{-6}m). Therefore:

$S_o = \pi r_o^2 = 0.7854$ μm^2 and $V_o = S_o.L_o = 1.7436$ μm^3 $(3.3)_{1\text{-}2}$

The maximum isometric force F_o could be calculated by replacing (3.1) and $(3.3)_2$ into:

$F_o = P_o.S_o = 176.715x10^{-9}N$ (3.4)

Burkholder and Lieber (2001, p.1531) superimposed data taken from 36 studies for bird, cat, rat, rabbit, mouse, frog, horse, human and derived a general normalized force-length relation for the sarcomer. Accordingly to their graph, the ascend limb of this relation is :

$$\begin{cases} 4.73k - 2.6156 for 0.572 \leq k \leq 0.736x \quad f = |1.185k - 0.122 for 0.736 \leq k \leq 0.947 (3.5) \\ \quad 1 for 0.947 \leq 1 \end{cases}$$

where $f = F/F_o$ and $k = L/L_o$. It has been shown (Pollack 1990, p.3; Epstein and Herzog,1998, p.76) that during shortening the length of the sarcomer linearly decreases :

$L(t) = At + B$ for $t_1 \leq t \leq t_s$ in units of μm (3.6)

Then the contraction speed is:

$v_z = du_z/dt = d(L_o - L(t))/dt = -A$ (3.7)

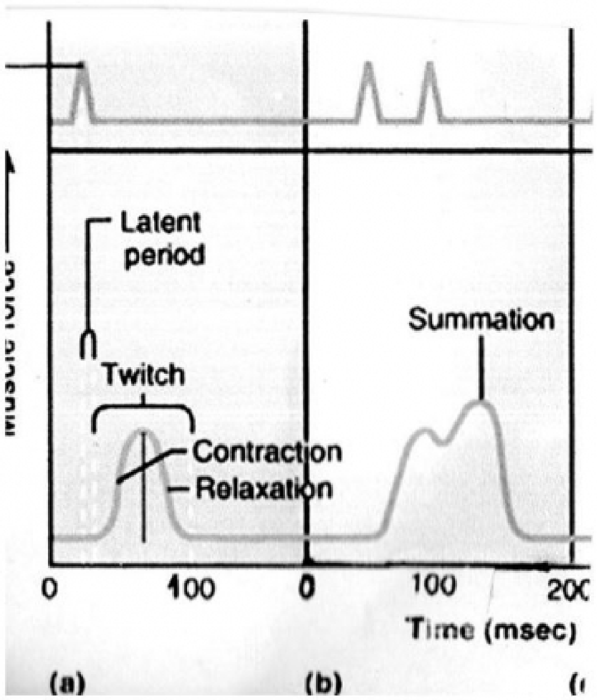
Faulkner et., al.,(1986) reported values for the contraction velocity 6 lengths/sec and 2 lengths/sec for fast and slow fibres respectively at temperature 37°C. We assume that we have to do only with fast fibres. Then from (3.7), it results:

$A = -6$ μm/sec (3.8)

In addition for fast- fibres $t_1 = 0.02$sec (see for example Makrorpoulou, 2009, p.16).

Figure 3

Electromyograph for muscle contraction. Taken from Makropoulou (2009)



Substituting the initial condition $L(t_1=0.02) = L_o= 2.22$μm into (3.6), it follows:

$L(t) = -6t + 2.34$ in units of μm (3.9)

Consequently by the help of (3.1), (3.2), (3.3), (3.4), (3.5) and (3.9) it is possible to calculate the normalized stresses, forces, lengths and volumes of the sarcomer, for various time moments. All results are concentrated in Table 1.Also Table 2. contains the results derived from linear regression analysis on data of Table 1. Particularly the forces and the lengths have been calculated by:

$f= F/F_o=(P.S_o)/(Po.S_o)= P/P_o =1.4- 6.933t$ and

$k= L/L_o = (V/S_o)/(V_o/S_o)=V/V_o = 1.0542-2.705t$ $(3.10)_{1-2}$

THEORETICAL RESULTS FOR CONTRACTION SPEED AND FOR NORMALIZED STRESS, FORCE, LENGTH AND VOLUME OF THE SARCOMER

In $(2.2)_3$ we restrict z to a constant z_1 such that:

$z_1= -0.00954x10^9$ (4.1) and we choose:

$a=-17x10^{-9}$ and $\theta=37^{\circ}C$ $(4.2)_{1-2}$

Table 1

All results for stresses, forces, lengths and volumes of the sarcomer, for various time moments of isokinetic contraction. The initial data are Po=225KN/m2, Fo=176.715Nx10−9, So =0.7854 μm2, Vo= 1.7436μm3 ,Lo = 2.22 μm and are picked from cites refered in the text.

P/P_o	F/F_o	L/L_o	V/V_o	t
1	1	1	1	t_1=0.02s
0.918	0.918	0.878	0.878	t_2=0.065s
0.892	0.892	0.856	0.856	t_3=0.073s
0.865	0.865	0.833	0.833	t_4=0.082s
0.839	0.839	0.811	0.811	t_5=0.09s
0.812	0.812	0.788	0.788	t_6=0.098s
0.786	0.786	0.766	0.766	t_7=0.107 s
0.758	0.758	0.743	0.743	t_8=0.115s
0.682	0.682	0.721	0.721	t_9=0.123s
0.577	0.577	0.698	0.698	t_{10}=0.132s
0.476	0.476	0.676	0.676	t_{11}=0.14s
0.371	0.371	0.653	0.653	t_{12}=0.148s
0.27	0.27	0.631	0.631	t_{13}=0.157s
0.165	0.165	0.608	0.608	t_{14}=0.165s
0.06	0.06	0.585	0.585	t_{15}=0.173s
0	0	0.572	0.572	t_{16}=0.178s

Table 2

All results for stresses, forces, lengths and volumes of the sarcomer de-rived by linear regression analysis F/Fo= 1.4−6.933t and L/Lo= 1.0542−2.705t on data of Table 1.

P/P_o	F/F_o	L/L_o	V/V_o	t
1.26	1.26	1	1	t_1=0.02
1.005	1.005	0.9	0.9	t_2=0.057s
0.893	0.893	0.857	0.857	t_3=0.073s
0.831	0.831	0.832	0.832	t_4=0.082s
0.776	0.776	0.811	0.811	t_5=0.09s
0.721	0.721	0.789	0.789	t_6=0.098s
0.658	0.658	0.765	0.765	t_7=0.107s
0.603	0.603	0.743	0.743	t_8=0.115s
0.547	0.547	0.721	0.721	t_9=0.123s
0.485	0.485	0.697	0.697	t_{10}=0.132s
0.43	0.43	0.676	0.676	t_{11}=0.14s
0.374	0.374	0.651	0.651	t_{12}=0.148 s
0.3115	0.3115	0.630	0.630	t_{13}=0.157s
0.256	0.256	0.608	0.608	t_{14}=0.165s
0.2	0.2	0.586	0.586	t_{15}=0.173s
0.166	0.166	0.573	0.573	t_{16}=0.178s

Replacing (4.1) and (4.2) into $(2.2)_3$ and $(2.3)_3$, it implies:

u_z =6t in units of μm and v_z=6μm/sec $(4.3)_{1\text{-}2}$

respectively. The theoretical result $(4.3)_2$ predicts the measurement of Faulkner et., al., (1986) for the contraction speed of fast fibres.

We normalize sarcomer's length:

k=L(t)/ L_o=L_o−u_z/ L_o=2.22−6t/ 2.22= (V/S_o) /(V_o/L_o)=V/V_o (4.4)

where we have used $(3.3)_2$ and $(4.3)_1$. At continuity we divide (2.6) with the maximum isometric stress (3.1) and we account that the coefficient az coincides with P_o (Tsili, 2013b. p.3). Then it implies:

T_{zz}/P_o = P/ P_o = c_{33}a□t/P_o + 1+a_{zz}a□/P_o (4.5) Hatta et., al. (1984) measured the elastic constant of muscle c33 in active state:

c_{33} =2,47x10□N/ m² (4.6)

and concluded that muscle elastically lies near an unstable state. Therefore more implicit situations such viscoelasticity are unstable. The last means that it is rather impossible to compute a standard value for the viscoelastic coefficient a_{zz}.

We only could predict the order of the magnitude of the a-bove parameter. We choose:

a_{zz} = −0.143x10^9N/m^2 (4.7)

Then (4.4) because of (3.1), $(3.2)_1$, (4.2), (4.6) and (4.7) concludes to:

P/P_o = 1.4−6.905t= (F/S_o)/ (F_o/S_o) = F/F_o (4.8)

Therefore using (4.4) and (4.8) it is possible to calculate all values for normalized stresses, forces, lengths and volumes for the sarcomer for the same time moments as we did earlier. All these results are found in Table 3.

DISCUSSION

Our model for proper choice of the unknown parameters, i) it predicts the contraction speed for fast fibres measured by Faulkner et., al., (1986) and ii) it is very closed to results for normalized stresses, forces, lengths and volumes of the sarcomer derived from statistical analysis on "raw" data (Table 2.), picked by Spector et., al., (1980); Roy et., al.,(1982); (1982); Powel et., al., (1984); Gordon et., al., (1966); Pollack (1990), Burkholder and Lieber (2001); Faulkner et., al., (1986), Epstein and Herzog (1998); Hatta et., al.,(1984); Makropoulou (2009). Compare our theoretical results in Table 3. with those of Table 2.

Table 3

The results for normalized stress ,force, length and volume of sarcomer derived from our model for a = −17x.10□9 , θο=37οC, c33= 2.47x109, azz=−0.143x109

P/P_o	F/F_o	L/L_o	V/V_o	t
1.262	1.262	0.946	0.946	t_1=0.02
1.006	1.006	0.846	0.846	t_2=0.057s
0.896	0.896	0.803	0.803	t_3=0.073s
0.834	0.834	0.778	0.778	t_4=0.082s
0.779	0.779	0.757	0.757	t_5=0.09s
0.723	0.723	0.735	0.735	t_6=0.098s
0.661	0.661	0.711	0.711	t_7=0.107s
0.606	0.606	0.689	0.689	t_8=0.115s
0.551	0.551	0.668	0.668	t_9=0.123s
0.489	0.489	0.643	0.643	t_{10}=0.132s
0.433	0.433	0.622	0.622	t_{11}=0.14s
0.378	0.378	0.6	0.6	t_{12}=0.148 s
0.316	0.316	0.576	0.576	t_{13}=0.157s
0.261	0.261	0.554	0.554	t_{14}=0.165s
0.205	0.205	0.532	0.532	t_{15}=0.173s
0.171	0.171	0.519	0.519	t_{16}=0.178s

References

1. Burkholder T. and Lieber R.(2001): Review. Sarcomer length operating range of vertebrate muscles during movement". J. Exper. Biology., 204, pp.1529-1536.
2. Faulkner J., Claflin D. and McCully K.(1986): "Power output of fast and slow fibers from human skeletal muscles, in human muscle power (eds N.L. Jones, N. Mc Cartney and A.J. Mc Co-mas), Human Kinetics Publishers, Champaign IL.
3. Gordon A. M., Hurxley A.F. and Julian F.(1966): "The variation in isometric tension with sarcomer length in vertebrate muscle fibers". J. Physiol. (London), 184:170-192.
4. Epstein M. and Herzog W. (1998): "Theoretical models of skeletal muscles". John Willey and sons Chichester, USA.
5. Hatta I., Hasegava, M., Nakayama H., et., al.,, (1984): " Ultra-sonic elastic constasnt of muscle".Jpn. J. Appl. Phys. Supple-ment 23-1, pp 66-68. Makropoulou M.(2009): "Biophysics of muscles" in: www.physics.ntua.gr/mmakro/index/Biofysics-Myes2009.pdf or www.zlab.rutgers.edu.classes/BehaviorCogNeuroBehavioral
6. Pollack G. (1990): "Muscles and molecules. Uncovering the principles of biological motion". Ebner and sons publications, USA.
7. Powel P., Roy R., Kanim P. et., al., (1984): "Predictabiliy of skeletal muscle tension from architectural determinations in quinea pig hindlimps".J. Appl. Physiol., Respirat. Environ. Exer-cise Physiol. 57(6):1715-1721.
8. Roy R., Meadows I.D., Baldwin K., et., al., (1982): "Functional significance of compensatory overload rat fast muscle".J. Appl. Physiol., Respirat., Environ. Exercize Physiol., 52, 473-478.

9. Spector S., Gardiner P., Zernicke R., et., al., (1980): “Muscle architecture and force velocity characteristics of cat soleus and medical gastrocnemius: implications for motor control”.J. Neurophysiol., 44., 951-960.
10. Tsili M. B. (2013a): “ Mechanics of muscle contraction. Part I. Theory.”Accepted by Internet Journal of Physiology and Pathophysiology , Quick Med Pub.
11. Tsili M. B. (2013b): “ Mechanics of muscle contraction. Part II. Application to isometric contraction”. Accepted by Internet Journal of Physiology and Pathophysiology , Quick Med Pub.

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