Internal Bone Remodeling Induced By The Distance-Running And The Unknown Remodeling Rate Coefficients.

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Abstract
We used the theory of adaptive elasticity (Hegedus and Cowin, 1976) and we qualitatively studied the internal remodeling of the tibia, induced by the distance-running. We showed that after a long time, an athlete’s tibia initially will be weaken and if the runner will not stop or decrease his (her) activity, eventually will be rupture. The result predicts “Tibial Stress Fracture” an overuse injury of the tibia (Kaplan et., al., 1997; Monaco et., al., 1997; Amendola et., al., 1999; Bouche, 1999; Walker, 1999; Jones, 2002; Romani, 2002; Mc Ginnis, 2005) due to suddenly increase in intensity and/or duration of training activity, participation to new one activity, exersizing to hard surfaces and poor footwear. After that we proved that only 12 possible combinations, for the sign of the five unknown coefficients $c_0$, $c_1$, $c_2$, $\alpha_T$ and $\alpha_A$ are mathematically predicted.

INTRODUCTION
Living bone is continually undergoing processes of growth, reinforcement and resorption, termed collectively “remodeling”. There are two kinds of bone remodeling: surface and internal (Frost, 1964). The internal bone remodeling capacity has been investigated by many authors (Martin, 1972; Cowin and Hegedus, 1976; Hegedus and Cowin, 1976; Cowin and Nachlinger, 1978; Cowin and Van-Buskirk, 1978; Carter et., al., 1987; Whalen, et., al., 1988; Carter et., al.,1989; Tsili, 2000, Qin and Ye, 2004; Tsili, 2009a).

The purpose of this work is to study the internal remodeling of the tibia, induced by the distance-running. For that reason we will use the theory of adaptive elasticity (Hegedus and Cowin, 1976).

BIOMECHANICAL ANALYSIS OF THE DISTANCE-RUNNING:
We assume that a person with bones which are not under osteoporosis or osteopetrosis (Hegedus and Cowin, 1976) starts distance-running activity and he (she) continues to be exercised, for a long time period. Initially the athlete was walking with a constant velocity $v_0$. Consequently the tibia was in a state at which no remodeling occurred, subjected only to a constant compressive load $G_0$, due to the vertical component of ground reaction force, at late stance phase during walking, given by:

$$G_0 = G_w - W_f \quad (2.1)$$

where $G_w$, $W_f$ are respectively: the vertical component of ground reaction force at late stance phase during normal walking and the weight of foot. Andriacchi et., al., (1977) and Rohrle et., al., (1984) used force-plate studies and evaluated the magnitude of the running load for certain magnitudes of the running velo-city. Selecting the above data and using a linear regression analysis, it is possible to find: (see also Whalen et., al., 1988)

$$G_w = 0.213v_0 + 0.913 \text{ in units of B.W.} \quad (2.2)$$

Also accordingly to Harless (1860):

$$W_f = 0.019B.W. \text{ in units of B.W.} \quad (2.3)$$

Then (2.1) because of (2.2) and (2.3) becomes:

$$G_0 = 0.213v_0 + 0.894 \text{ in units of B.W.} \quad (2.4)$$

Accordingly to Whalen et., al., (1988) the walking activity corresponds to a velo-city whose magnitude lies in the range [0.5m/sec, 2m/sec], starting with slow and ending with fast walking. Then from (2.4) it implies that:
1.0005B.W. < G o < 1.32B.W. (2.5)

Also initially the tibia had a uniform volume fraction $\xi_0$, 0 < $\xi_0$ < 1 and a uniform relative volume fraction $e_0 = 0$.

At $t = 0$ the athlete starts running activity as it seems in Fig. 1, taken from Clisouras (1984). Accordingly to Clisouras (1984), runners are classified into three categories: sprinters, middle-distance and distance-runners. The last category contains: the cross-country and track runners who participate to 5000m, 10000m and marathon race of men/women, as well new soldiers or students of military Academies, who follow their basic training. At late stance phase during running the foot of the sprinter, middle distance-running and distance-running, contacts the ground: with the toes (forefoot strikers), the second third of the foot (middlefoot strikers) and the heel (rearfoot strikers) respectively (Cavanagh and LaFortune, 1980; Clisouras, 1984). The abovementioned styles of running seem in Figs. 2a, 2b and 2c (see also Tsili, 2009b). Also accordingly to Clisouras speaking for distance-running we mean that the athlete/new soldier/student of military Academy runs with a constant velocity. Then his/her tibia is under a constant axial load $G_z$, due to the vertical component of ground reaction force, at late stance phase during running (see Fig. 2c) such that:

$$G_z = G_z f - W_f$$ (2.6)

where $G_z f$ is the vertical component of ground reaction force at late stance phase for the foot during running.

**Figure 1**
Fig 1.: An athlete is distance-running (From Clisouras, 1984).
Alexander and Jayes (1980); Bates et., al., (1983); Cavagna (1964); Cavagna and La-Fortune (1980); Fukanaga et., al., (1980); Winter (1983) used force-plate studies and computed the magnitude of the running load, for certain magnitudes of running velocity. Selecting the above data and using a linear regression analysis, it is possible to obtain (see also Whalen, et., al., 1988):

\[ G_{zf} = 0.46v + 0.55 \text{ in units of B.W} \] \hspace{2cm} (2.7)

while \( v \) is the running velocity. Then (2.6) because of (2.3), (2.7) becomes:

\[ G_z = 0.46v + 0.531 \text{ in units of B.W} \] \hspace{2cm} (2.8)

We will study the dynamic problem of a hollow circular cylinder, consisting of a material which behavior is described by the theory of adaptive elasticity (Hegedus and Cowin, 1976). The cylinder has an inner and outer radii: \( a \) and \( b \) respectively and a length \( L \). These radii are constants, since in present work we deal only with the internal remodeling (Frost, 1964; Hegedus and Cowin, 1976; Cowin and Van-Buskirk, 1978; Cowin and Van-Buskirk, 1979).
where $B$ is athlete’s body weight and assumed to be constant during the training period.

Accordingly to Cowin and Nachlinger (1978), our problem has a unique solution. Assume the followings:

$$u = A(t)r + B(t)/r$$
$$u = 0$$
$$u = C(t)z$$

where $A(t)$, $B(t)$, $C(t)$ are unknown functions. Then (3.2) becomes:

$E_r = A(t) - \frac{B(t)}{r^2}$
$E_{rr} = A(t) + \frac{B(t)}{r^2}$
$E_{zz} = C(t)$
$E_z = 0$
$E_r = 0$

and

Consequently the stress-strain relations (3.4) are now written as:

$T_{rr} = 2(\lambda_1 + \mu_2)A(t) - \frac{B(t)}{r^2} + \lambda_2 C(t)$
$T_{rr} = 2(\lambda_1 + \mu_2)A(t) + \frac{B(t)}{r^2} + \lambda_2 C(t)$
$T_{rr} = 0$
$T_{rr} = 0$

Applying the boundary conditions, it is possible to find that:

$$A(t) = \lambda_3 \frac{G_f B}{2(\lambda_1 + \mu_2)}$$
$$C(t) = -\frac{G_f B}{2(\lambda_1 + \mu_2)}$$

where:

$$\gamma = (\lambda_1 + 2\mu_2)\lambda_3$$

The displacements, the strains and the stresses can now be calculated by replacing (3.12)–(3.13) into eqns. (3.9), (3.10) and (3.11) respectively. When the obtained strains are employed, the remodeling equation (3.1) takes the form:

$$e = A(e) + \frac{\lambda_3 G_f B}{\pi(b-a)}$$

Since we don’t know the exact values of the material functions, we will use approximate forms of them, for small values of $e$. Accordingly to Hegedus and Cowin (1976); Cowin and Van- Buskirk (1978), the proper approximations are:

$$A(e) = c_0 + c_1 e + c_2 e^2$$
$$A(e) = \lambda_3 + e\lambda_3$$
$$\lambda_1 = \Lambda_1 + e\Lambda_1$$
$$\lambda_2 = \Lambda_2 + e\Lambda_2$$
$$\mu_1 = \mu_1 + e\mu_1$$
$$\mu_2 = \mu_2 + e\mu_2$$

where $c_0, c_1, c_2, \ldots$ are constant coefficients. Particularly $\mu_1$. 

Figure 6

The equations of the theory of adaptive elasticity (Hegedus and Cowin, 1976) are:

$$A(t) + A(t) + A(t) + A(t)$$

(3.1)

the strain-displacement relations

$$E_r = u_v$$
$$E_{rr} = u_{rr}$$
$$E_{zz} = u_{zz}$$

and

$$E_{zz} = \frac{1}{2} u_{rr} v_{rr}$$

(3.2)

and the stress equations

$$T_{rr} + \frac{1}{r} T_{rr} - \frac{1}{r} T_{rr} + \frac{1}{r} T_{rr} = p_{ij}$$

(3.3)

Figure 7

The stress-strain relation for a transversely isotropic elastic material are:

$$T_{rr} = (\lambda_1 + \mu_2)E_r + \lambda_2 E_{rr} + \lambda_3 E_{zz}$$
$$T_{rr} = \mu_2 E_{rr}$$
$$T_{rr} = \lambda_2 E_{zz}$$
$$T_{rr} = \lambda_2 E_{zz}$$

and

$$T_{rr} = 2\mu_2 E_r$$

(3.4)

Figure 8

where

$$\lambda_1 = \frac{\nu_r E_r - 2\nu_v E_r}{(1-2\nu) E_r}$$
$$\lambda_2 = \frac{2\nu_v E_r}{(1-2\nu) E_r}$$
$$\mu = \frac{E_r}{1+\nu}$$

In addition the boundary conditions for our problem are:

$$T_{rr} = T_{rr} = 0$$

at $r = a$ and $z = 0$

(3.6)

$$T_{rr} = T_{rr} = 0$$

at $r = b$ and $z = 0$

(3.7)

$$T_{rr} = -\frac{G_f B}{\pi(b-a)}$$

at $a < b$ and $z = 0$

(3.8)
$\Lambda_2$, $\Lambda_1$ and $\Lambda_2$ are known (Cowin and Van-Buskirk, 1978), while the rests are termed as “remodeling rate coefficients” of the theory of adaptive elasticity (Hegedus and Cowin, 1976) and have not been experimentally determined. Therefore (3.14) after the above approximations, concludes to:

**Figure 13**

\[
\dot{e} = \dot{c}_t e^2 + c_0 e + c_0 + \frac{[\Lambda_1 a_x - (\Lambda_2 + \Lambda_3) a_x] G B}{\pi (b^2 - a^2) F} \tag{3.16}
\]

Thus we deal with an equation of the form:

\[
\dot{e} = \alpha (e^2 - 2\beta e + \gamma) \tag{3.17}
\]

where:

**Figure 14**

\[
\begin{align*}
\alpha &= c_1, \\
\beta &= \frac{-c_1}{2c_2}, \\
\gamma &= \frac{1}{c_1} \left( \frac{[\Lambda_1 a_x - (\Lambda_2 + \Lambda_3) a_x] G B}{\pi (b^2 - a^2) F} \right)
\end{align*}
\]

and

\[
F = (\Lambda_2 + \Lambda_3)(\Lambda_1 + 2\Lambda_2) - \Lambda_1^2 \tag{3.18}
\]

The mathematical analysis, the exploration of the solutions of (3.17) that satisfy the initial condition $e(0) = 0$ and the final results, are in my earlier work (see Tsili, 2000, pp.237-238) and they will be repeated here. We only remind that:

**Figure 15**

\[
e_i = \beta + \sqrt{\beta^2 - \gamma} \quad \text{and} \quad e_o = \beta - \sqrt{\beta^2 - \gamma} \tag{3.19}
\]

**DISCUSSION**

Our model predicts “tibial stress fracture” an overuse injury of tibia (Kaplan et al., 1997; Monaco et al., 1997; Amendola et al., 1999; Bouche, 1999; Walker, 1999, Romani, 2002; Jones, 2002; McGinnis, 2005), due to suddenly increase in intensity and/or duration of training activity, participation to new one activity, exercising to hard surfaces and poor footwear (Kaplan et al., 1997; Monaco et al., 1997; Amendola et al., 1999; Bouche, 1999; Walker, 1999; Jones et al., 2002; Romani, 2002). “Stress fracture” occurs commonly among marchers, distance-runners and new soldiers or students of military Academies, during their basic training (Foster, 1899; Bernstein et al., 1946; Brubaker and James, 1974; McBrady, 1975; Gudas, 1980; Orava, 1980; Taunton et al., 1980; Bensel and Kish, 1983; Sullivan et al., 1984; Hulkko and Orava, 1987; Matheson et al., 1987; Markley, 1987; Greany et al., 1987; Hahn et al., 1991; Kaplan et al., 1997; Monaco et al., 1997; Amendola et al., 1999; Romani et al., 2002).

Tibial stress fracture occur among persons with normal bones who are undergoing physical activity to which they are unaccustomed (Devas, 1969; Devas, 1970; Belkin, 1980; Jones et al., 2002). Increased or different activity results in an altered relationship of bone growth and repair (Wolff law). When remodeling predominates over repair, the cortex transiently weakens and if stress continues, eventually ruptures (Walker, 1999). The resulting stress may run the spectrum from a microfracture to rupture of bony cortices with a fracture line (Markley, 1987; Walker, 1999).

In the early stages of stress fracture, bone scans are negative. As the stress fracture begins to mature, typical findings show i): subperiosteal resorption and occasionally a small fracture line for the posteromedial area of tibia and ii): “the dreaded black line” of the horizontal fracture, for the anterior area of this bone (Monaco et al., 1997; Walker, 1999) as it seems in Fig. 3. Therefore the acceptable solutions of our problem are in Table 1. Accordingly to this table after a long time, the tibia of the runner initially will be weaken. At continuity if the athlete will not temporally interrupt or decrease his (her) running activity, the tibia eventually will be ruptured.

**Figure 17**

Table 1.:The acceptable solutions of our problem.

\[
\begin{align*}
\text{Case:} & \\
\text{The solutions and their physical sense:} & \\
\text{For} & \quad t < \tau = \lim_{t \to \infty} (t < \tau) & \text{The tibia has been weakened} \\
\text{For} & \quad t = \tau = \lim_{t \to \infty} (t = \tau) & \text{The tibia has been healed} \\
\text{For} & \quad t > \tau = \lim_{t \to \infty} (t > \tau) & \text{The tibia has been healed} \\
\text{For} & \quad t < \tau = \lim_{t \to \infty} (t < \tau) & \text{The tibia has been weakened} \\
\end{align*}
\]

**RESULTS**

As we stated earlier, in the remodeling rate equation (3.16) there are 5 unknown coefficients $c_0$, $c_1$, $c_2$, $\ldots$, $\ldots$. 

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Consequently there are $3^5 = 243$ possible cases, concerning their signs. Employing the initial condition $e(0) = 0$ into (3.16), it is possible to obtain:

**Figure 19**

If $c_0 = 0$, then from (5.1) it results that $\Lambda_1 \alpha \tau - (\Lambda_2 + \mu_2) \alpha \Lambda = 0$. Consequently the rate remodeling equation (3.16) concludes to the form:

$$\ddot{e} = c_2 e + c_1 e \quad (5.2)$$

and its meaning is that: bone remodeling process is not depended upon the applied stresses, or with other words the mechanical loads do not affect the bone remodeling process. The last contradicts to the law of Wolff (1884; 1892) and therefore $c_0 \neq 0$.

We distinguish two cases, for the type of the solutions of Table 1.

1) If the first or the third solutions of Table 1 hold, then from (3.18)1 we obtain that $c_2 > 0$. Also from (3.19)1-2 it is easy to find that $\gamma < 0$ . Then (3.18)3 because of (3.18)1 and (5.1) results to:

$$[\Lambda_1 \alpha \tau (\Lambda_2 + \mu_2) \alpha \Lambda] (G_f - G_o)B / \pi (b^2 - a^2)F < 0 \quad (5.3)$$

Accordingly to Cowin and Van- Buskirk (1978, p. 274):

$$\Lambda_1 = 40G \alpha \Lambda_2 = 40G \alpha \mu_1 = 7G \alpha \mu_2 = 4G \alpha \quad (5.4)$$

1-2-3-4

Consequently from (3.18) 4 and (5.3) we conclude that:

$F > 0$ and $[\ddot{e} - (\Lambda_2 + \mu_2) \alpha \Lambda] R < 0 \quad (5.5)$

where:

Accordingly to Whalen et., al., (1988) the walking and running activities correspond to velocities whose magnitudes are of the order of: $v_o = 1.25 \text{m/sec}$ and $v = 4.5 \text{m/sec}$ respectively. Then from (2.4) and (2.8) it is possible to conclude that $G_o$ and $G_f$ are of the order of $1.1 \text{B.W.}$ and $2.60 \text{B.W.}$ respectively.

Then from (5.6) we obtain that $Q > 1.501$ and $R > 0$, while (5.5) 2 by the help of (5.6) 1 becomes:

$$\ddot{e} < 1.1 \dot{e} \quad (5.7)$$

Then from (5.1) and (5.5) 1 we find that $c_0 > 0$. Also (5.7) because of (5.4) 1-2-4 concludes:

$$\ddot{e} < 1.1 \dot{e} \quad (5.8)$$

The parameters $\ddot{e}$ and $\dot{e}$ might both be positive or negative, or it might $\ddot{e} < 0$ and $\dot{e} > 0$. The case $\ddot{e} = 0$ is excluded, because from (3.15) 3 it follows that the material function $A_A (e)$ vanishes. The last contradicts to the fact that the bone is a transversely isotropic material (Reilly and Burstein, 1975). Table 2. contains all possible combinations, for the sign of the unknown remodeling rate coefficients.

**Figure 21**

<table>
<thead>
<tr>
<th>$c_0$</th>
<th>$c_2$</th>
<th>$\Lambda_1$</th>
<th>$\Lambda_2$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0&lt;0$</td>
<td>$c_2&lt;0$</td>
<td>$\Lambda_1&lt;0$</td>
<td>$\Lambda_2&gt;0$</td>
<td>$\mu_1&gt;0$</td>
<td>$\mu_2&gt;0$</td>
<td>$R&gt;0$</td>
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<tr>
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<td>$c_2&lt;0$</td>
<td>$\Lambda_1&lt;0$</td>
<td>$\Lambda_2&lt;0$</td>
<td>$\mu_1&lt;0$</td>
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<td>$R&lt;0$</td>
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<td>$\Lambda_1&gt;0$</td>
<td>$\Lambda_2&gt;0$</td>
<td>$\mu_1&gt;0$</td>
<td>$\mu_2&gt;0$</td>
<td>$R&gt;0$</td>
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<tr>
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<td>$c_2&gt;0$</td>
<td>$\Lambda_1&gt;0$</td>
<td>$\Lambda_2&lt;0$</td>
<td>$\mu_1&lt;0$</td>
<td>$\mu_2&lt;0$</td>
<td>$R&lt;0$</td>
</tr>
</tbody>
</table>

**Table 2:** All the possible cases for the sign of correspond to the first or to the forth type of solutions of Table 1.

ii) If the second or forth solutions of Table 1. hold, then from (3.18)1 it implies that $c_2 > 0$. From (3.19) 2 it is possible to obtain that $\ddot{e} < 0$ and $\dot{e} > 0$. The first because of (3.18) 2 gives that $c_1 > 0$, while the second by the help of (3.18) 1 , (3.18) 3 and (5.1) concludes to:
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\[
\begin{align*}
\{(l \in \mathbb{R} (a + 2) : l \in \mathbb{R}) & | \text{G} f \rightarrow G o \} / (b 2 - a 2) : f > 0 \text{(5.9)} \\
\text{The last by the help of (5.5) } l \text{ leads to :}
\end{align*}
\]

\[
\begin{align*}
\{(l \in \mathbb{R} (a + 2) : l \in \mathbb{R}) & | R > 0 \text{(5.10)} \\
\text{where } R \text{ is again defined from (5.6) } l \text{. Then (5.10) gives us that:}
\end{align*}
\]

\[
\begin{align*}
\{(l \in \mathbb{R} (a + 2) : l \in \mathbb{R}) & | R > 0 \text{(5.11)} \\
\text{Then from (5.1) it implies that } c o < 0. \text{ Also (5.11) because of (4.1) 1-2-4 con-}
\end{align*}
\]

cludes that:

\[
\begin{align*}
\{(l \in \mathbb{R} (a + 2) : l \in \mathbb{R}) & | T > 1.1 \text{(5.12)} \\
\text{The parameters } l \text{ and } T \text{ might be both positive or negative or it might be } T > 0 \text{ and } A < 0. \text{ Table 3. contains all possible combinations for the sign of the unknown coefficients. Accordingly to the results of Tables 2. and 3., totally 12 cases for the sign of } c o, c 1, c 2, l 1, l 2, l 3, l 4 \text{ are theoretically predicted. Thus the i-nitial number of 243 mathematically possible cases, is dramatically restricted.}
\end{align*}
\]

References

46. Tsili M.C. B. (2009b): “Surface bone remodeling induced by the distance running and medial tibial stress syndrome (shin splints)”.
51. Wolff J. (1892): “Das gesetz der transformation knochen”. Hirswald Berlin
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