The Golden Proportion: Key To The Secret Of Beauty
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Citation

Abstract
The Golden Proportion, a mathematical ratio, represents beauty, harmony and balance in physical form. Over the centuries, this geometric constant has influenced architecture, biological systems, mathematics and art. This ratio is believed to hold the key to the secret of beauty and finds its representation in innumerable natural and manmade masterpieces. The paper discusses various aspects of this ratio and their relevance in human aesthetics.

INTRODUCTION
The Golden Proportion (...) has been known as mathematics of harmony since antiquity. This is believed to be a blueprint for features in nature, art, architecture and humans that conform to harmony and beauty [...]. This proportion encompasses both organic and inorganic entities and has been found represented in numerous natural and architectural marvels ranging from Egyptian Pyramids, famous Greek temple Parthenon, classical work of Leonardo Da Vinci “Mona Lisa” and “Last Supper”, Corbusier human body sketch of proportion “Le Modulor”, musical compositions of Mozart, Beethoven to the human form itself.

The Golden Proportion is defined geometrically as the ratios, where the ratio of the whole segment to the longer segment is equal to the ratio of the longer segment to the shorter segment. Mathematically, the precise value of this Ratio is expressed as 1.6180339887..., a never-ending, never-repeating number which goes to infinity. Hence this ratio cannot be expressed as a whole number or as a fraction and is considered an irrational number. Drawing algebraically this ratio, the point C divides the line AB in such a way that the ratio of AC to CB is equal to the ratio of AB to AC. The algebraic calculation shows that the ratio of AC to CB and AB to AC equals 1.618... while the ratio of CB to AC is equal to 0.618...

Expressing Golden proportion (Φ) using a line segment, the ratio of total length A + B is to the longer segment A is equal to ratio of A to the shorter segment B.

Expressed algebraically:

Figure 1

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Expressed algebraically:

Mathematically, these ratios are such that the longer segment is 1.618054 times the length of the shorter segment, while the shorter is 0.618054 times the longer. This astonishing number is the only one in mathematics which, when subtracted by units (1.0), yields its own reciprocal. The progression using the golden proportion numbers is unique and remarkable because, without exception, the proportion of the mathematical ratio of 0.618... to 1.000 is universal; 1 divided by 0.618... equals 1.618.Conversely, 0.618... divided by 1 equals 0.618...[5,6] This unique proportion in thus known as golden proportion (Φ) and is a universal principle which occurs on all levels of creation.

This fraction has influenced the artists, musicians,
The concept of Golden Proportion was well known to ancient Greeks and had tremendous influence on their art and architecture. The majority of the ancient Greek buildings, including the Parthenon, considered being antiquity's most perfect structure was constructed upon the principle of the Golden Proportion. The Greeks attributed the discovery of golden proportion to Pythagoras (560-480 BC), who was a great Greek geometer of the 5th century B.C. He showed that the human body is built with each part in a definite golden proportion to all the other parts. A little later, Euclid, a great Greek mathematician (365BC - 300BC) defined the same as a proportion derived from a division of a line at the 0.6180399... into what he called its “extreme and mean ratio” in his book “Elements”. Euclid's mentions: “A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.” [9]

The Golden Rectangle, an embodiment of the Golden Ratio, is also made on the same principle and was also used frequently in Greek architecture. The Golden Rectangle’s ratio of the height to width equals the Golden Ratio 1.618... which makes it one of the most visually satisfying geometric forms. [Figure 3]

This has the unique property that when a square is removed (square 1 in the diagram below) smaller rectangles of the same shape remains. Then again if a smaller square is removed (2), a smaller rectangle of the same shape remains, again a process that seemingly could go on indefinitely. All the squares are in Golden relationship, with each quadrant being 1.618... times longer than the one preceding it. A ‘golden spiral’ phi (Ф) can also be generated by linking the
quadrants (quarter circles) of each square of the Golden Rectangle.

The Phidias, a famous Greek sculptor and mathematician used the Golden Proportion in his architecture so much that it came to be known as phi (ф). He sculptured many architectural masterpieces including Parthenon which was built in about 440BC at the Temple of Athena on the Acropolis in Athens. The Parthenon's facades as well as elements of its facade were built in the golden rectangles with the altitude constructed in the proportion as 1.0 and the base 1.618 times the altitude. The spaces between the columns also formed the golden rectangles. [10] [Figures 4, 5]

Figure 6
Figure 4. Parthenon

Figure 7
Figure 5. Golden proportions in Parthenon

During the Hellenistic period, the human body was considered as one of the most perfect example of symmetry and eurhythmy. [11] Doryphoros statue created by Polycleitus, [Figure 6] considered an architectural marvel of the classic Greek sculpture, is also made on the same principle.

The influence of Golden Proportion secret declined with the fall of Greece, but it began to resurface in the 15th century when Classical Renaissance artists namely Michelangelo, Raphael, Van Gogh, Leonardo da Vinci began using the Golden Proportion in their paintings and sculptures to achieve beauty and balance. Leonardo da Vinci (1451-1519) is believed to have incorporated Golden Proportion in his well-known paintings namely the “Vitruvian Man”, “The Last Supper” and the classical masterpiece “Mona Lisa”. In “Vitruvian Man” (or Man in Action) he used the Golden Proportion to create illustrations for the mathematician Luca Pacioli’s paper “De Divina Proportione” (1509). This was probably the earliest reference of the Golden Proportion as the “Divine Proportion”. The drawing shows a nude man inscribed in a circle with arms and legs outstretched like spokes in a wheel [Figures 7, 8].
The distance from the top of the man’s head to the middle of his chest is 1.618… times the length of the head alone. The distance from the top of his head to his navel is 1.618… times the distance from his head to the middle of the chest, and so on.

Leonardo da Vinci also used Golden Proportions in his classical painting “The Last Supper” starting from the dimensions of the table at which Christ and his disciples sat to the proportions of the walls and windows in the background. [Figure 9]

His masterpiece marvel “Mona Lisa” is also said to have the golden ratios in its geometric equivalents. The evaluation of the painting reveals many golden rectangles. The rectangle around her face is golden and further division of that rectangle with a line drawn across the eyes, another golden rectangle is obtained, making the proportion of her head length to her eyes as golden. If a rectangle is drawn whose base extends from the her right wrist to her left elbow and if we extend the rectangle vertically until it reaches the very top of her head we again get a golden rectangle. If we draw squares inside this Golden Rectangle, we will discover that the edges of these new squares come to all the important focal points of her face: chin, eye, nose, and the mouth. [Figure 10, 11]
Leonardo Da Vinci also incorporated the Divine Proportion in the design of famous cathedral Notre de Dame in Paris. [Figure 12]

Le Corbusier (1887–1965), the famous French architect and painter developed a scale of proportions which he called “Le Modulor”, based on a human body whose height is divided in golden section commencing at the navel. He then subdivided those sections in golden ratio at the knees and throat. [Figure 13]
The concept was supposed to provide a standardized system that would automatically confer harmonious proportions applicable to man, mechanics and architecture. He described the concept of the golden ratio as “rhythms apparent to the eye and clear in their relations with one another. And these rhythms are at the very root of human activities. They resound in man by an organic inevitability, the same fine inevitability which causes the tracing out of the Golden Section by children, old men, savages and the learned.”

The concept also find its representation in world renowned monument “Taj Mahal” [Figure 14,15] constructed in 1648 by the Mughal Emperor Shahjahan, in Agra, India and also in modern architecture marvels like United Nations building in New York [Figure 16], The CN Tower in Toronto[Figure 17] and “Golden Ratio” sculpture in Jerusalem.[Figure 18]
Figure 16
Figure 15. Golden proportions in Taj Mahal

Figure 17
Figure 16. Golden proportions in United Nations building
Figure 18
Figure 17. Golden proportions in CN Tower
Evidence suggests that the Golden proportion also presented in classical musical compositions written by Mozart, Beethoven and Bach. Mozart divided a striking number of his sonatas into two parts whose lengths reflect the golden proportion. The first movement of Mozart's sonata consists of 100 measures that are divided into the customary two parts; 38 in the first, 62 in the second making this ratio (38/62) 0.613 which gets closer to 0.618 in a composition of 100 measures. The second movement of this sonata is also divided according to the Golden proportion. Whether it was a conscious consideration or just coincidence, the mystery persists.\[15\]

The representation of golden proportion is also abounding in nature and animal kingdom. Probably nature also prefers certain angles and ratios in order to optimize its creations. There are numerous examples of this perfect ratio from the pentagonal symmetry of the flowers to the logarithmic spiral of the chambered nautilus shell.\[16,\]\[17\] The nautilus builds a shell to protect itself from the outside elements. As it further grows, it builds another chamber at the shell bigger than the preceding one, and after moving into this bigger area closes off the previous. It continues this process of building larger and larger chambers along a logarithmic spiral. The resultant spiral inside the shell is a Golden Proportioned spiral with space of each successive chamber having approximately 1.618 times more volume than the previous chamber. [Figure 19].\[18\]

The proportions of different plant components including the ideal distribution and positioning of leaf around a stem and diameters of geometrical figures inside the flowers often shows the Golden Proportion or angle in several species.\[19\] The phyllotaxis is based on a Golden angle of approximately 222.5 degrees rotated from one leaf to the next, which probably provides the flower petals and leaves with maximum sun exposure and allows rain drops to flow down to the root in the most efficient manner. [Figure 20]
It is known as the Golden angle as it divides the 360 degrees circle into a Golden Ratio: 222.5/360 = 0.618055… and 360/222.5 = 1.617977… The individual florets of the sunflower also grow in two spirals extending out from the centre. The first spiral contains 24 arms, while the other contains 35, making the ratio 24 to 35 a Golden Ratio. [Figures 21, 22]

**Figure 22**
Figures 21, 22. Sunflower florets

The spirals of a pinecone, where spirals from the center have 5 and 8 arms, respectively [or 8 and 13, depending on the size] again represents Fibonacci sequence. [Figures 23, 24]

In animal kingdom there are plenty examples of the Golden proportion being represented in dolphin, butterfly moth to the peacock’s feather. [Figure 25, 26]

**Figure 24**
Figure 25. Butterfly moth
In a dolphin's body, the eye, fins and tail all fall at golden sections of its length. The eye-like markings of the butterfly falls at golden sections of the lines that mark its width and length.[Figure 27] The ratio of the lengths of the thorax and abdomen in most bees is nearly the golden ratio and the peacock feather’s presents 12 interdependent Golden Proportions. 

The DNA, the basic molecule of life also contains Golden Proportion in his structure. The cross section of the DNA double helix forms a golden decagon which is a constituent of two pentagons rotated by 36 degrees from the other having the diagonal ratios of 1:1.618. The DNA molecule measures 34 angstroms long by 21 angstroms wide for each full cycle of its double helix shape making a ratio of 1.619 (ratio 34/21 equals 1.619…), which is very close to 1.618. The ratio of the major to the minor groove (21 angstroms to 13 angstroms) in DNA is also golden (21/13 equals 1.619). [Figure 27]

In Physics of atoms, despite of four fundamental asymmetries namely, structure of atomic nuclei, distribution of fission fragments, distribution of numbers of isotopes, and the distribution of emitted particles “the numerical values of all of these asymmetries are equal approximately to the golden ratio. The electrons also follow Fibonacci sequence with changing states of hydrogen atoms during the change of orbits. 

In the universe there are many spiral galaxies representing the golden ratio in their structures. [Figure 28]
The orbital relationships between certain planets are also golden proportional. The distance from Mercury to Venus being approximately 1.618 times the distance from the Sun to Mercury. The distance from Earth to Mars being approximately 1.618 times the distance from Venus to Earth. There is an emergent theory which suggests that the universe may actually be in the shape of a dodecahedron based on the Golden Proportion. Johannes Kepler, (1571-1630) the discoverer of the elliptical nature of the orbits of the planets around the sun, described the Golden Proportion as: “Geometry has two great treasures: one is the other, the division of a line into extreme and mean ratio. The first we may compare to a measure of gold; the second we may name a precious jewel.”

No consideration of the golden proportion however can be complete without the mention of the great mathematician of the middle ages, Leonardo Pisano Fibonacci (1170–1250). In the 12th century he discovered a mathematical series that find its representation throughout the nature. The classical problem of “rabbits reproduction” is one of the most known among the many problems formulated by Fibonacci resulting in the discovery of the numerical sequence of 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610... to infinity, known as Fibonacci numbers. [Figure 29]
The height of the human body in comparison to the distance from the head to the hand is 1.618. The ratio of the forearm to the hand is 1.618. The combined length of the hand and the forearm divided by the length of the forearm results in 1.618. Similarly, the ratio of upper arm + forearm to hand + forearm is in the same ratio of 1: 1.618. The ratio of successive phalanges of the digits and the metacarpal bone also approximates the golden ratio of 1.618.24

The golden proportion finds its representation in innumerable ancient Greek canons and defines the relationships between various areas of the head and face. [11],[23] [Figures 31, 32]

**Figure 30**
Figure 31. Greek canon

The head forms a golden rectangle with the eyes at its midpoint. The width of the face is a golden section of the length of the face beginning from the top of the head to the menton. The mouth and nose are placed at the golden sections of the distance between the eyes and the menton. In the golden ratio taken from the total facial height, the ratio of the eye to menton is 1.618. A reverse measurement from menton to the ala of the nose is golden to the forehead. The width of the nose is golden to the width of the mouth and the eyes are golden to the mouth. The head width at the temple is golden to the eyes’ width. Given the upper lip length from ala to mouth as 1.0, the eye to the ala of the nose is golden to it and the mouth to chin is golden to it. With reference to the distance from eyebrow to ala of nose the cheek prominence is located at the 1.618 relationship. The lateral canthus of the eye is also golden to the eyebrow to the cheek prominence. The width of the Cupid’s bow peaks is golden to the distance from one peak to either commissure. [6] The human ear also represents the golden rectangle with its architecture closely resembling the shape of a Fibonacci spiral. The cochlea in the inner ear too has a logarithmic spiral shape containing the golden proportion. [Figure 33]
The ideal dentition also represents the golden proportion. The frontal four teeth from central incisor to premolar, the most significant part of the dental aesthetics are in Golden Proportion to each other. [Figure 34]

The ratio of the width of the central incisor is in Golden ratio to the width of the lateral incisor. The width of the lateral incisor is Golden to the width of the canine and the width of the canine is Golden to that of the first premolar creating a rhythmic normal occlusion. The height of the central incisor is in the Golden Proportion to the width of the two central incisors. [Figure 35]. [6], [24] Many studies done to investigate the relation between the golden proportion and dental aesthetics have varying opinions. Many studies have been reported in literature echoing golden proportion in dental aesthetics. However they have varying opinions regarding the relation between the golden proportion and dental aesthetics.

DISCUSSION

From time immemorial, the enigma of beauty exists. The beauty is defined as a characteristic that provides a pleasurable perceptual experience to the eye and human brain. The most debatable issue is what makes a person beautiful? Is beauty really in the eye of the beholder or is it determined by some objective parameters governed by a mathematical ratio? Since antiquity, artists and scientists have tried to quantify the form of the ideal or most perfect and beautiful face using subjective and objective criterias which are well represented in ancient Greek canonicals. Subjective factors because of the large diversity in perception of beauty in different cultures varied from continent to continent and country to country. Some of the women and men being considered attractive in one geographical area are not considered the same in the other. Additionally, extensive variability in the human face, experience, insight and personal values also plays an important role in assessment. Hence subjective factors are not enough to define ideal beauty and the objective analysis of the face is imperative. In practical it is an important step in the approach to the patient desirous of improved facial aesthetics. Various objective methods of determining ideal facial aesthetics have been discussed in the literature ranging from cephalometric / anthropometric analysis, photogrammetry, application of ideal golden mask [Figure 36], computer simulations, optical-surface scanning and 3-D facial scan [Figure 37] and imaging but are not conclusive. [34], [35], [36], [37], [38], [39], [40]

However they represent one thing in common - symmetry and proportion. Hence even though the enigma of ideal facial beauty persists, the Golden Ratio remains the foundation for all the past, present and future facial analysis systems. [41]. The principle needs to be applied in conjunction with other factors including prevalent racial and cultural characteristics, respect for the patient's individuality and surgical possibilities to get the desired result.

The Golden Proportion is a fascinating ratio which appears to be a universal constant of harmony and beauty, with its creation being represented throughout the universe ranging from nature, art, architecture and even human form itself. Why is this particular ratio and its representations seem more appealing to the human eye and mind? Is this the Nature's perfect number? Is this the language of the Universe? We do not have any answer yet. But undoubtedly this mathematical law quantitatively defines beauty, balance and harmony, the understanding of which is an essential prerequisite for achieving aesthetically pleasing results.

SUMMARY

Beauty is a quality which gives pleasure to the senses and is characterized by balance of proportions. The “Golden ratio” which gets succinctly expressed in the ratio of the number “1” to the irrational “1.6180339887...” represents ideal measured relationships and encourages a scientific appreciation of symmetry and beauty and has a definite impact on human aesthetics. Further understanding of this concept with its possible recreation in human aesthetics might help in achieving what we always dream… the beauty beyond perfection.

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