Mechanics Of Muscle Contraction Part III: Isokinetic Shortening Contraction

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Citation

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Abstract
In this paper we applied a proposed theory (Tsili, 2013a) and we investigated the isokinetic shortening contraction of the sarcomer. We assumed that during contraction, the change of its cross-section area was negligible. Our model for special values of parameters, predicts the contraction speed measured by Faulkner et., al., (1986) at temperature 37°C. Also our findings that deal with normalized stress, force, length and volume of the sarcomer, are very close to the results derived from statistical analysis on “raw” data taken from Gordon et., al., (1966); Spector et., al., (1980); Roy et., al., (1982); Powell et., al., (1984); Faulkner et., al., (1986); Pollack (1990); Epstein and Herzog (1998); Burkholder et., al., (2001); Makropoulou, (2009).

INTRODUCTION
The purpose of this paper is to study the problem of isokinetic shortening contraction of the sarcomer. For that reason we will base upon a recently developed theory (Tsili, 2013a). The main assumption of this work is that during contraction the change of cross-section area of the sarcomer is small and could be neglected.

THE PHYSICAL APPROXIMATION OF THE PROBLEM
The basic equations of our theory are in our previous paper (Tsili, 2013a) and they will not be repeated here. We supposed that at t<0, the sarcomer was free of stress. In resting state it had a length $L_0$, a radius $r_0$ and a temperature $\theta_0$, as indicated in Fig. 1. Consequently its cross-section area and initial volume of sarcomer were $S_0$ and $V_0$, respectively, where:

\[ S_0 = \pi r_0^2 \quad \text{and} \quad V_0 = S_0 L_0 \]  

We assume that at t=0 the motor neuron sends a stimulative pulse to the motor unit, in order to start the process of muscle contraction. Due to the delay of the response, at t1>0 a mechanical stress is produced. The last simultaneously displaces with constant speed upwards and downwards the material particles of the sarcomer that lie below and above the z line respectively. As a result of the above displacements, the sarcomer shortens (see Fig. 2.).
process of shortening continues until a time $t_s$. At $t > t_s$ the contraction stops and the inverse process of relaxation starts.

**Figure 2**
The sarcomer at time $t \geq t_1$, during shortening. Accordingly to all that we stated in previous paragraph, the displacements are: $u_r = u_\theta = 0$ and $u_z = a \theta z$ (2.2)1-2-3

Accordingly to all that we stated in previous paragraph, the displacements are:

$$u_r = u_\theta = 0 \quad \text{and} \quad u_z = a \theta z \quad (2.2)1-2-3$$

where $a$ is an unknown constant. Therefore the velocity of the material particle of sarcomer is:

$$v_r = du_r/dt = 0 \quad v_\theta = du_\theta / dt = 0 \quad \text{and} \quad v_z = du_z / dt = a \theta z \quad (2.3)1-2-3$$

Then strain–displacement equations become:

$$E_{rr} = E_{\theta\theta} = E_{r\theta} = E_{z\theta} = E_{rz} = 0 \quad \text{and} \quad E_{zz} = a \theta z \quad (2.4)1-2-3-4-5-6$$

Substituting the above into stress - strain equation (see Tsili, 2013a) it follows that all stresses vanish:

$$T_{rr} = T_{\theta\theta} = T_{r\theta} = T_{z\theta} = T_{rz} = 0 \quad (2.5)1-2-3-4-5$$

except from the axial stress:

$$T_{zz} = c_{33} a \theta z + a_s + a_a \theta \quad (2.6)$$

**CALCULATION OF STRESS, FORCE, LENGTH AND VOLUME OF THE SARCOMER, USING “RAW” DATA**


$$P_o = 225 \text{KN/m}^2 \quad (3.1)$$

Accordingly to Gordon et., al., (1966) and to Pollack (1990), the initial length and diameter of sarcomer are:

$$L_o = 2.22 \mu m \quad \text{and} \quad d_o = 2r_o = 1 \mu m \quad (3.2)1-2$$

Therefore:

$$S_o = \pi r_o^2 = 0.7854 \mu m^2 \quad \text{and} \quad V_o = S_o L_o = 1.7436 \mu m^3 \quad (3.3)1-2$$

The maximum isometric force $F_o$ could be calculated by replacing (3.1) and (3.3) into:

$$F_o = P_o S_o = 176.715 \times 10^5 \text{N} \quad (3.4)$$

Burkholder and Lieber (2001, p.1531) superimposed data taken from 36 studies for bird, cat, rat, rabbit, mouse, frog, horse, human and derived a general normalized force-length relation for the sarcomer. Accordingly to their graph, the ascend limb of this relation is:

where $f = F/F_o$ and $k = L/L_o$. It has been shown (Pollack 1990, p.3; Epstein and Herzog, 1998, p.76) that during shortening the length of the sarcomer linearly decreases:

$$L(t) = At + B \quad \text{for} \ t \leq t_s \quad \text{in units of } \mu m \quad (3.6)$$

Then the contraction speed is:

$$v_z = du_z / dt = d(L_o - L(t)) / dt = -A \quad (3.7)$$

Faulkner et., al.,(1986) reported values for the contraction velocity 6 lengths/sec and 2 lengths/sec for fast and slow fibres respectively at temperature 37°C. We assume that we have to do only with fast fibres. Then from (3.7), it results:

$$A = -6 \mu m/sec \quad (3.8)$$

In addition for fast- fibres $t_1 = 0.02$sec (see for example Makropoulou, 2009, p.16).
Substituting the initial condition \( L(t_1=0.02) = L_o = 2.22 \mu m \)
into (3.6), it follows:

\[
L(t) = -6t + 2.34 \text{ in units of } \mu m \quad \text{(3.9)}
\]

Consequently by the help of (3.1), (3.2), (3.3), (3.4), (3.5) and (3.9) it is possible to calculate the normalized stresses, forces, lengths and volumes of the sarcomer, for various time moments. All results are concentrated in Table 1. Also Table 2. contains the results derived from linear regression analysis on data of Table 1. Particularly the forces and the lengths have been calculated by:

\[
f = F/F_o = (P/S_o)/(P_o/S_o) = P/P_o = 1.4 - 6.933t \quad \text{(3.10)}
\]

\[
k = L/L_o = (V/S_o)/(V_o/S_o) = V/V_o = 1.0542 - 2.705t \quad \text{(3.10)}
\]

**THEORETICAL RESULTS FOR CONTRACTION SPEED AND FOR NORMALIZED STRESS, FORCE, LENGTH AND VOLUME OF THE SARCOMER**

In (2.2), we restrict \( z \) to a constant \( z_1 \) such that:

\[
z_1 = 0.00954 \times 10^9 \quad \text{(4.1)}
\]

and we choose:

\[
a = 17 \times 10^{-9} \text{ and } \theta = 37^\circ C \quad \text{(4.2)}
\]

Replacing (4.1) and (4.2) into (2.2) and (2.3), it implies:
\[ u_z = 6t \text{ in units of } \mu \text{m and } v_z = 6\mu \text{m/sec} \quad (4.3) \]

respectively. The theoretical result (4.3) predicts the measurement of Faulkner et. al., (1986) for the contraction speed of fast fibres.

We normalize sarcomer’s length:

\[ k = \frac{L(t)}{L_o} = \frac{u_z}{L_o} = 2.22 \times \frac{6t}{2.22} = \left( \frac{V}{S_o} \right) \left( \frac{V_o}{L_o} \right) = \frac{V}{V_o} \quad (4.4) \]

where we have used (3.3) and (4.3). At continuity we divide (2.6) with the maximum isometric stress (3.1) and we account that the coefficient \( a_z \) coincides with \( P_o \) (Tsili, 2013b, p.3). Then it implies:

\[ \frac{T_{zz}}{P_o} = \frac{P}{P_o} = \frac{c_{33}}{a_z} \theta_0 + \frac{a_{zz}}{P_o} \quad (4.5) \quad \text{Hatta et. al. (1984) measured the elastic constant of muscle } c_{33} \text{ in active state:} \]

\[ c_{33} = 2.47 \times 10^9 \text{ N/m}^2 \quad (4.6) \]

and concluded that muscle elastically lies near an unstable state. Therefore more implicit situations such viscoelasticity are unstable. The last means that it is rather impossible to compute a standard value for the viscoelastic coefficient \( a_{zz} \).

We only could predict the order of the magnitude of the above parameter. We choose:

\[ a_{zz} = -0.143 \times 10^{-9} \text{N/m}^2 \quad (4.7) \]

Then (4.4) because of (3.1), (3.2), (4.2), (4.6) and (4.7) concludes to:

\[ \frac{P}{P_o} = 1.46 \times 9.05t = \left( \frac{F}{S_o} \right) \left( \frac{F_o}{S_o} \right) = F/F_o \quad (4.8) \]

Therefore using (4.4) and (4.8) it is possible to calculate all values for normalized stresses, forces, lengths and volumes for the sarcomer for the same time moments as we did earlier. All these results are found in Table 3.

**DISCUSSION**

Our model for proper choice of the unknown parameters, i) it predicts the contraction speed for fast fibres measured by Faulkner et. al., (1986) and ii) it is very closed to results for normalized stresses, forces, lengths and volumes of the sarcomer derived from statistical analysis on “raw” data (Table 2.), picked by Spector et. al., (1980); Roy et. al.,(1982); (1982); Powel et., al., (1984); Gordon et., al., (1966); Pollack (1990), Burkeholder and Lieber (2001); Faulkner et., al., (1986), Epstein and Herzog (1998); Hatta et., al.,(1984) and Makropoulou (2009). Compare our theoretical results in Table 3. with those of Table 2.

**Table 3**

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**References**


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