Modeling Life Expectancy for Aged Male and Female Population of Bangladesh: Regression Model Approach

R Islam

Citation

Abstract
The purpose of the present study is to construct statistical models to life expectancy at exact age x \( (e_x) \) for aged male and female population of Bangladesh. And also study differentials, patterns and trends of aged life expectancy at exact age x \( (e_x) \) for male and female population of Bangladesh. For that rationale, the secondary data of life expectancy at exact age x \( (e_x) \) for aged male and female population of Bangladesh have been used in this study. To prove the soundness of the model, the model validation technique, cross-validity prediction power (CVPP), is applied. It is seen that simple linear regression model is fitted to life expectancy for aged male and female population of Bangladesh with explaining more than 98% proportion of variation and more than 97% stability of each model.

INTRODUCTION
In developing countries like Bangladesh, population is called aged who belongs to the age group 60 years and above. But, in developed countries the population is said to be aged who belongs to the age group 65 years and over. In life table analysis, the expectancy of life is the function most frequently used as an index of the level of mortality. The life expectancy at ages also represents the summarization of the complete chain of mortality rates for all ages combined, as weighted by the life table stationary population. In particular, the life expectancy at age 60 years is normally used as the measure of mortality at the older ages, the ages where most of the deaths are occurred in the developing as well as underdeveloped countries. Government, Non-Government organizations and other life insurance companies are needed to obtain up to date information of life expectancy for the older ages of population. Hope this study would be helpful to bolster their plan especially for aged population for the concerned organizations.

It was observed that the pattern or shape of the distribution of life expectancy at exact age x \( (e_x) \) for female population of Bangladesh follow biquadratic polynomial model (Islam, 2006a). Islam (2006b) found that the distribution of age associated with life expectancy \( (e_x) \) for male population of Bangladesh follow 4\(^{th}\) degree polynomial model but in which the term containing x is excluded because of the insignificant of the coefficient of x. But, in this study, an effort has been given concentration that which types of model or functional relationship exists between age and the life expectancy at exact age x \( (e_x) \) for aged male and female population of Bangladesh.

Therefore, the fundamental aims and objectives of the current study are to build up statistical models to life expectancy for aged male and female population of Bangladesh, and also study differentials, patterns and trends of aged life expectancy for male and female population of Bangladesh.

This paper is structured as follows. The data sources of this study are presented in section 2. Section 3 describes the methodology in which regression model fitting, model validation procedure and F-test are included. Numerical results of model fittings and discussion of this paper are reported in section 4. Finally, section 5 concludes the conclusion of the manuscript.

DATA SOURCES OF THIS STUDY
To complete the aforesaid objectives of this paper the secondary data on life expectancy at exact age x \( (e_x) \) for aged male and female population of Bangladesh are taken from (Islam, 2006c, 2007a and 2007b). These are utilized as raw resources of the present study that are shown in Table 1 for male and Table 2 for female.
METHODOLOGY

REGRESSION MODEL FITTING

Using the scattered plot of aged life expectancy at exact age \( x \) (\( e_x \)) for male and female population of Bangladesh, it appears from the Figure 1 to Figure 6 that these are linearly distributed. So, a statistical model, that is, a simple linear regression model is measured and the structure of the model is

\[
y = a + a_1 x + u \quad \text{(Montgomery and Peck, 1982)};
\]

where, \( x \) represents age group (in years); \( y \) represents aged life expectancy (\( e_x \)); \( a, a_1 \) are unknown parameters and \( u \) is the stochastic disturbance term of the model.

It is to be noted that these models are built using the software STATISTICA.

MODEL VALIDATION PROCEDURE

To check the validity or legality of these models, the CVPP, is employed here. The mathematical formula for CVPP is given below:

\[
\rho_{cv}^2 = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)} (1-R^2).
\]

Where, \( n \) is the number of classes, \( k \) is the number of explanatory variables in the model and the cross-validated \( R \) is the correlation between observed and predicted values of the dependent variables (Stevens, 1996). The shrinkage coefficient of the model is the positive value of \(-R^2\); where is CVPP and \( R^2 \) is the proportion of variation of the fitted model. Additionally, 1-shrinkage coefficient is the stability of \( R^2 \) of the model. The estimated CVPP analogous to their \( R^2 \) of the model are summarized in Table 3. It was informed that CVPP was also employed by Islam (2006a and 2007b), Islam et al (2003) and Khan and Ali (2004) as model rationalization method.

F-TEST

To detect the overall measure of significant level of the fitted models as well as the significance of \( R^2 \), the F-test is performed in this manuscript. The F-test is specified by

\[
F = \frac{R^2}{1-R^2} \quad \text{with (l-1, n-1) degrees of freedom (d.f.);
}
\]

where \( l = \) the number of parameters is to be estimated in the fitted model, \( n \) is the number of cases and \( R^2 \) is the coefficient of determination of the model (Gujarati, 1998).

4. Numerical Results of Model Fittings and Discussion

The statistical models, that is, simple linear regression model is assumed to fit to age related life expectancy for aged male and female population of Bangladesh and the fitted models are presented in the first column of Table 3. The findings on model fittings and estimated CVPP corresponding to their \( R^2 \) of these models are shown in Table 3. It appears from this table that the fitted models (1) - (6) are highly cross- validated and their shrinkage coefficients are shown in the sixth column of the same table. These imply that the fitted models (1) - (6) will be stable more than 97% that are demonstrated in the fifth column of the Table 3. Moreover, it is found that the parameters of all the fitted models are highly statistically significant with more than 98% of variance explained that is also indicated in Table 3. The stability for \( R^2 \) of these models is more than 99%.

The calculated values of F statistic for all the fitted models with (1, 4) d.f. are displayed in the same table whereas the analogous tabulated values are only 21.2 for (1) – (6) models at 1% level of significance. As a result, from these statistics it might be concluded that these models and their analogous \( R^2 \) are highly statistically significant. Hence, the fits of all these six models are well.

It should be mentioned here that other as usual models, for instance, exponential, quadratic, cubic, log-linear were also tried to fit model to these data aggregate but those are not fit well due to significant of parameters, shrinkage coefficients and proportion of variation.
Figure 3
Life Expectancy at Exact Age x (e) for Aged Male Population of Bangladesh During 2005-2007

<table>
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<td>14.82</td>
<td>16.09</td>
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Figure 4
Life Expectancy at Exact Age x (e) for Aged Female Population of Bangladesh During 2005-2007

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Figure 5
The Results of CVPP and Information on Model Fittings

<table>
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<tr>
<th>Models</th>
<th>n</th>
<th>K</th>
<th>R²</th>
<th>ϕp²</th>
<th>Shrinkage</th>
<th>Cal F</th>
<th>Tab. F (5%, n_k)</th>
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<td>a) y=x+0.2153-0.572x p-value=0.0000 (0.0000)</td>
<td>6</td>
<td>1</td>
<td>0.99915</td>
<td>0.979</td>
<td>0.0103</td>
<td>585.95</td>
<td>21.2</td>
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<tr>
<td>b) y=x+0.1258-0.328x p-value=0.0000 (0.0000)</td>
<td>6</td>
<td>1</td>
<td>0.99733</td>
<td>0.995</td>
<td>0.0029</td>
<td>464.95</td>
<td>21.2</td>
</tr>
<tr>
<td>c) y=x+0.0215-0.232x p-value=0.0000 (0.0000)</td>
<td>6</td>
<td>1</td>
<td>0.99733</td>
<td>0.994</td>
<td>0.0026</td>
<td>444.24</td>
<td>21.2</td>
</tr>
<tr>
<td>d) y=x+0.0125-0.137x p-value=0.0000 (0.0000)</td>
<td>6</td>
<td>1</td>
<td>0.99733</td>
<td>0.992</td>
<td>0.0013</td>
<td>270.28</td>
<td>21.2</td>
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</table>

Figure 6
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Male Population of Bangladesh in 2005. X axis represents Age Group and Y axis represents Male Life Expectancy.

Figure 7
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Male Population of Bangladesh in 2006. X axis represents Age Group and Y axis represents Male Life Expectancy.
**Figure 8**
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Male Population of Bangladesh in 2007. X axis represents Age Group and Y axis represents Male Life Expectancy.

**Figure 9**
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Female Population of Bangladesh in 2005. X axis represents Age Group and Y axis represents Female Life Expectancy.

**Figure 10**
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Female Population of Bangladesh in 2006. X axis represents Age Group and Y axis represents Female Life Expectancy.

**Figure 11**
Observed and Predicted Life Expectancy at Exact Age x (e) for Aged Female Population of Bangladesh in 2007. X axis represents Age Group and Y axis represents Female Life Expectancy.

**Figure 12**
Trend of Life Expectancy at Exact Age x (e) for Aged Male Population of Bangladesh During 2005-2007. X axis represents Age Group and Y axis represents Male Life Expectancy.
Figure 13
Trend of Life Expectancy at Exact Age x (e) for Aged Female Population of Bangladesh During 2005-2007. X axis represents Age Group and Y axis represents Female Life Expectancy.

Furthermore, from the figures it is observed that the pattern of aged life expectancy for male and female population of Bangladesh are downward due to ages at every year of the study period and almost all the same pattern are displayed to see in the graph paper. But, the trends of these are upward, that is, increasing at every ages with passing of time for male and female population. Moreover, it is also investigated that female life expectancy is greater than that of male at every ages excepting the last age groups during the study period.

CONCLUSION
In this study it is found that age associated with aged life expectancy for male and female population of Bangladesh follow simple statistical linear regression model. The up to date information on aged male and female life expectancy of Bangladesh would be helpful to life insurance companies to bolster their plan especially for the aged population.

References
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Author Information

Rafiqul Islam
Associate Professor and Ex-Chairman, Dept. of Population Science and Human Resource Development, University of Rajshahi