
Density Incidence And Cumulative Incidence: A Fundamental Difference

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Citation

P Philippe. *Density Incidence And Cumulative Incidence: A Fundamental Difference*. The Internet Journal of Internal Medicine. 2000 Volume 2 Number 2.

Abstract

This methodological note expands on the relation of density incidence (DI) to cumulative incidence (CI). DI is a force and is thereby seen as a property of the risk factor while CI, as a frequency, is a property of the persons getting diseased. Emphasizing the difference between both concepts allows one to better understand why they are irreducible to each other and lend themselves to differential epidemiologic/public health uses.

INTRODUCTION

A few years ago concepts of density incidence (DI) and cumulative incidence (CI) were still confounded in epidemiology textbooks. Some authors have suggested distinguishing them clearly. It is important to differentiate both concepts because they are design-specific [although both types of measures can actually be computed in a given design] and their meaning is in general irreducible. The current paper aims at emphasizing the difference between both concepts and points out the implications of this difference for epidemiologic and public-health purposes.

In this paper, the words: probability, proportion, and risk are equivalent (see the glossary). All represent concepts related to a closed-cohort design. In contradistinction, the word rate has to do with the concepts of force, intensity, density, and speed, and is related to the open-cohort design. Further, in this paper, computation of the rate is for recurrent events. Though this practice is not habitual in epidemiology (the analysis of non-recurring events or first-occurring events being a particular case of the general computation of rates), to consider the rate from the standpoint of recurrent events has three major advantages:

- The approach is general rather than specific;
- It better conveys the fundamental distinction between a rate and a risk;

- The rate of recurring events has an important meaning in epidemiology that is sometimes overlooked.

Both the concepts of DI and CI apply to cohort studies. There are two basic cohort designs: the open cohort and the closed cohort. The open cohort allows entries and losses to follow-up. The open cohort, therefore, involves turnover of its population. Individuals as such have no particular meaning in this design as they can be substituted for others. A useful model for the open cohort is that of a disco. Let us suppose an observer whose task is to evaluate cigarette smoking during a given period of time. Individuals enter and quit (losses to follow-up) the disco at different times and remain in the disco for different time periods during which they smoke. What is of concern to the observer is cigarette smoking (not smokers); therefore, the turnover of the observed population is immaterial as long as individuals of the population are interchangeable and that cigarette smoking is doomed to stay constant during the observation period. The undertaking aims at providing an estimate of the intensity of cigarette smoking during the observation period.

A closed cohort, on the other hand, allows losses to follow-up only. And, in contradistinction with the open cohort in which individuals enter and quit the disco at any time, the closed cohort defines itself by the group of individuals present at the outset of the observation period and,

accordingly, allows losses to follow-up only. An appropriate model of the closed-cohort design would be a party where all guests arrive at the same time while some leave the group during the evening with no attempt to comeback later on. An observer that would be present from the start might be interested in providing an estimate of the frequency of smokers.

DENSITY INCIDENCE

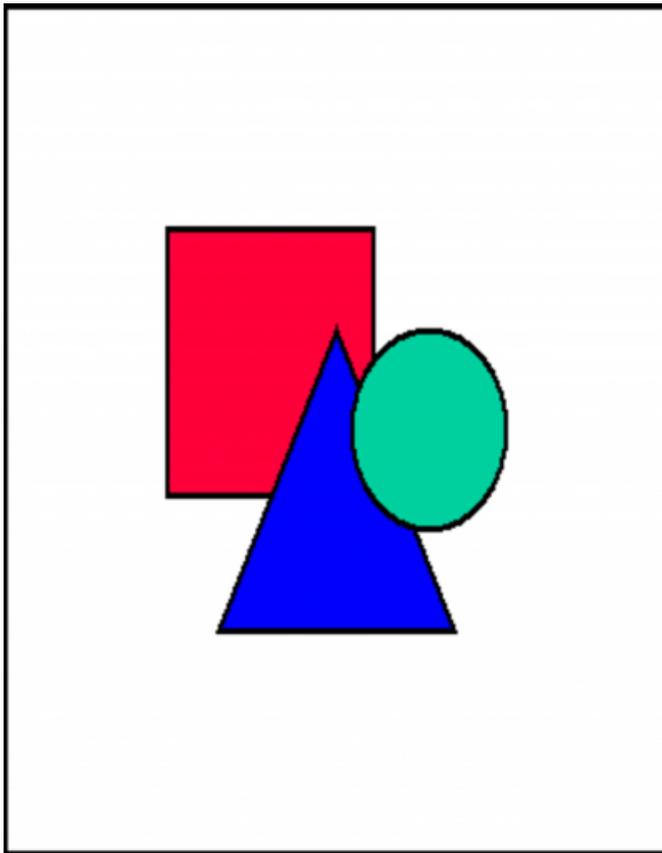
DI is defined as the ratio of incident cases to the population at risk in the course of a time period. This definition is more operational than meaningful. While the numerator of the ratio results from a count of events (the number of cigarettes smoked in the disco or the number of pathological events during an observation period), the denominator of DI is the sum of the time periods contributed by the individuals (at risk of the event in the numerator). It is clear from the above definition that DI represents an intensity/density (or force) as the number of incident events is reported to a sum of time periods (rather than to persons as such) acting as a sole (long) individual time period. The ratio of the numerator to the denominator thus acts as a measure of the «concentration» of observed events (density) by individual and time period. Therefore, if one counts 250 cigarettes smoked by 100 individuals in the disco in the course of one hour, this is equivalent to say that the 250 cigarette count boils down to observing one individual during 100 hours (subjected to the above hypotheses as to the stability of the rate and interchangeability of the individuals). Or, in other words, that one person smokes a mean of 2.5 cigarettes per hour. Clearly, this is a speed. It can also be viewed as a force acting on the average individual. In a different context, when one asserts that the number of yearly cancer cases is equal to 250 (new diagnoses) in an at-risk population of 100,000, one stresses the fact that the (cancer) risk factors operate on an individual (equivalent to all others at risk) with a force of 0.00250 per year. Thus, DI appears to be a property of the risk factors. This is clear when one thinks of DI as a measure of force acting over an average individual at risk. Therefore, DI measures the inducing potential of the risk factors to cause disease. DI thus has no relation to individuals as such, and does not measure the frequency of diseased individuals. The interest rate is a case in point here; the interest rate represents the force with which a given capital grows in the course of a time period. The interest rate does not specify the amount of money earned, only the force that operates on the capital. The interest rate also shows that the fundamental dimension of the rate is the time without which the rate is meaningless. In the context of enzymatic transformations, DI

is also viewed as the speed of transformation of a substrate into a given metabolite under enzyme activity during a given time period. To sum up, DI is a measure of speed, force, density, intensity, and is a property of the inducing factor of the observed event. DI is therefore a rate that cannot be confounded with a proportion or probability. As a force, DI is related to causal research. In Newtonian physics a force induces a movement; analogically DI engenders diseased individuals the frequency of which is measured by CI.

CUMULATIVE INCIDENCE

CI is simply defined as the ratio of incident cases to the at-risk population at the beginning of the observation period (when losses to follow-up are non-existent). The denominator of CI is not time anymore nor any sum of contributed time periods; it is rather the number of individuals at risk. CI thus determines a frequency that measures the proportion of individuals getting diseased, or the proportion of smokers in a party if one is willing to take up the model suggested at the outset of this paper. One will note the fundamental difference between DI and CI; DI measures the intensity of cigarette smoking in the disco while CI measures the frequency of smokers in a party. Even though the DI of non-recurring events may also be a measure of the frequency of individuals, it remains that DI is fundamentally the force of a phenomenon that takes its meaning from the general case of recurring events. DI is therefore a property of the risk factors (or the force that acts upon individuals at risk) while CI concerns the individuals themselves. Similarly, the speed of a car is a property of its engine, while the frequency of road accidents may be viewed as a consequence of the speed of the car. In epidemiologic terms, one would say that the force of risk factor elicit diseased individuals as cigarette smoking determines smokers. Things go as though both concepts would measure different dimensions of the same reality. For instance, in the case of non-recurring events, one can go from the force of the risk factor (DI) to the number of diseased individuals (CI) through the negative exponential function such that $CI = 1 - \exp(-DI)$. Thus, a force of .2/yr. acting constantly in a pool of 1000 at-risk individuals followed up for one year will produce 181 diseased individuals from the outset. Thus, things go as though the two measures belonged to different compartments as in the accompanying Image.

Figure 1



From the Image, one notes:

- that risk factors determine DI;
- that CI characterizes individuals, thus measuring their transition from health to disease;
- and that both compartments can be related by a simple relationship [$IC = 1 - \exp(-ID)$] in the case of non-recurring events (undefined in the recurring case).

CONCLUSION

DI and CI are two irreducible concepts that find applications in specific cohort designs (although both measures can be computed in a given design). The consequence is that a rate and a probability are not interchangeable. As a measure of the force of the inducing risk factor, DI has no individual meaning. The rate measures the inducing force of the risk factor to generate diseased individuals (the etiologic function of the rate), the order of magnitude of which is measured by

the proportion of diseased individuals (the preventive function of the proportion). The proportion measures individual risks and is limited to 100% while the rate, as a speed, bears no such limit.

Interestingly, studying recurring events rather than first-occurring such events (or once-occurring events) emphasizes the fundamental opposition between a rate and a risk. Studying recurring events thus allows one to characterize more meaningfully DI and CI. Although epidemiology deals more often with non-recurring or first-occurring events, the fundamental difference between the two measures ought to be stressed. This difference is less obvious in the case of once-occurring events. In the latter case, the rate can conceptually be confounded with the risk.

GLOSSARY

A proportion is the ratio of two quantities, the denominator of which contains the numerator. Multiplied by 100, a proportion can vary between 0 and 100%.

A probability differs from a proportion to the extent that one wishes to stress the difference between a population and a sample. In this case, the proportion refers to an estimate of the parameter belonging to the population. The proportion tends to the probability to the extent that the sample size gets closer to the population size. A probability varies between 0 and 1.

The risk is the subjective expression of the probability. In usual terms, the perception of risk is highly personal. The risk evokes the chance of an event existing or occurring. To the extent that the probability has been derived from the survey of an existing population, it estimates the risk of finding a (diseased) case; to the extent that the probability has been computed from a followed-up population, one rather speaks of the risk of an event occurring.

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